Nonlocal dynamical response of a ballistic nanobridge

2010 J. Phys.: Condens. Matter 22 025304

(http://iopscience.iop.org/0953-8984/22/2/025304)

IOP PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER
J. Phys.: Condens. Matter **22** (2010) 025304 (9pp) doi:10.1088/0953-8984/22/2/025304

Nonlocal dynamical response of a ballistic nanobridge

V L Gurevich, V I Kozub and M I Muradov

A F Ioffe Institute, Russian Academy of Sciences, 194021 Saint Petersburg, Russia

Received 16 September 2009, in final form 18 November 2009
Published 14 December 2009
Online at stacks.iop.org/JPhysCM/22/025304

Abstract
The nonlocal dynamical response of a ballistic nanobridge to an applied potential oscillating with frequency ω is considered. It is shown that, in addition to the active conductance, there is also a reactive contribution. This contribution turns out to be inductive for relatively small frequencies ω. For bigger frequencies the current response is either inductive or capacitive, depending on the ratio of $\omega L/v_F$, where L is the length of the bridge and v_F is the Fermi velocity.

1. Introduction

Starting in the 1970s point contacts and nanobridges have been widely used for studies of elementary events related to electron transport. Sharvin constriction [1] or a classical point contact between two bulk conductors is realized when the width of the constriction is reduced so that it is smaller than the electron mean free path. In particular, these devices allow studies of elementary scattering events related to electron–phonon scattering and scattering of electrons by single defects (point-contact spectroscopy, see, e.g., [2]).

Nanobridges with a width comparable to the Fermi wavelength have been found to conform to the Landauer conductance quantization theory [3] (see [4, 5] for a review). Such a nanobridge can be formed by means of negatively biased metallic gates on top of a semiconductor heterostructure confining a two-dimensional electron gas. Thus the understanding of details related to frequency-dependent electron transport in these devices is of undoubted interest.

The dynamical response of ballistic structures was first considered by Kulik *et al* [6] for a classical ballistic point contact between two bulk metallic banks. It was shown that, in addition to an active contribution to the impedance, there also exists a reactive contribution of inductive nature. The latter is called kinetic inductance. The purpose of this paper is to investigate this phenomenon in a 1D ballistic structure where the lateral quantization is of importance, namely a ballistic nanobridge. To begin with, we wish to indicate that an estimation of a kinetic inductance of a 1D bridge cannot be realized without taking into account the dispersion of the bridge 1D conductance G. To do this, let us at first disregard the dispersion. Then the additional energy of electrons acquired in the bridge under the bias V is

$$W \sim \frac{1}{2}(eV)^2 \frac{\partial n_3}{\partial \mu} L\mathcal{A}, \tag{1.1}$$

where n_3 is the 3D electron concentration, so that $n_3 L\mathcal{A}$ is the total number of electrons within the bridge, L is the length of the bridge and $\mathcal{A}$ is the area of its cross section. We have taken into account that only electrons within the energy layer of a thickness eV near the Fermi level μ are accelerated by the bias. Having in mind that

$$n_3 = \frac{p_F^3}{3\pi^2\hbar^3}, \qquad \frac{\partial n_3}{\partial \mu} = \frac{m p_F}{\pi^2\hbar^3} \tag{1.2}$$

(m is the effective mass) we can write

$$W \sim \frac{1}{2}\frac{e^2}{G^2}\frac{m p_F}{\pi^2\hbar^3} L\mathcal{A} J^2, \tag{1.3}$$

where J is the current and G is the bridge conductance. This energy can be associated with a kinetic inductance $\mathcal{L}_k$ as

$$W = \frac{1}{2}\frac{\mathcal{L}_k J^2}{c^2}. \tag{1.4}$$

As is well known, due to transverse quantization the electron spectrum in 1D systems is represented by a number of quantum channels, i.e. minibands, with the dispersion law $\varepsilon_n(p_x)$ (n being the number of a channel). One can rewrite G in terms of a number of quantum channels $\mathcal{N}$ as

$$G = G_0\mathcal{N} \qquad \text{where } \mathcal{N} = \mathcal{A}p_F^2/4\pi\hbar^2 \tag{1.5}$$

and

$$G_0 = e^2/\pi\hbar. \tag{1.6}$$

G_0 has units of velocity. We have

$$G_0 = 6.96 \times 10^7 \text{ cm s}^{-1}. \tag{1.7}$$

Thus finally we obtain

$$\mathcal{L}_k \sim \frac{e^2c^2}{G^2}\frac{mp_F}{\pi^2\hbar^3}L\mathcal{A} \sim \frac{4c^2}{Gv_F}L, \tag{1.8}$$

where $v_F = p_F/m$. In particular, for $L = 10^{-4}$ cm, $\mathcal{N}$ = 10 and $v_F = 10^6$ cm s^{-1} we have $\mathcal{L}_k$ about 500 cm. To estimate the limits of observability of $\mathcal{L}_k$ one should compare the inductive resistance $\omega\mathcal{L}_k/c^2$ with the dissipative resistance G^{-1}. As is seen, their ratio becomes much larger than unity for frequencies

$$\omega \gg \frac{v_F}{L} \tag{1.9}$$

that, for the values of the parameters above, give $\omega/2\pi$ bigger than ∼10 GHz. Physically this condition means that the frequency $\omega/2\pi$ should be larger than the reciprocal electron time of flight. For the frequencies satisfying equation (1.9) one should take into account the dispersion[1] of conductance G. In other words, to treat the kinetic inductance we have to work out a theory of nanobridge conductance $G(\omega)$.

We will treat the situation where the nanobridge is surrounded by other conductors, including, in the first place, the split gates, which suppress electric fields outside the nanostructure. This is a principal difference from the situation considered by Sablikov and Shchamkhalova in [7] where no conductors outside the nanobridge were presumed, except plane electrodes at the contacts of the bridge. Below we will discuss the difference between our results and those of [7].

We will see that, because of the electron–electron (e–e) interaction, the screening radius a enters the equations describing the spatial distribution of the electric field and current. It is interesting to note that this is the case provided the radius of screening is smaller than the lateral width b of the conductor. In the opposite case, where $a \gg b$, b plays the role of the screening radius. This situation is analyzed in detail in appendix B.

Based on the studies of [7, 8] one could expect that the results obtained by these authors including, in particular, the reactive impedance, are generically of a quantum nature. We believe, however, following Kulik *et al*, that the physics behind the reactive component of the impedance in the case considered is of a purely classical nature. We will show this for the dynamical inductance. In this paper we take into consideration the effect of lateral quantization but otherwise formulate the problem on the basis of a classical approach. Such an approach will permit us to derive an equation for the impedance $Z(\omega)$ covering the whole classical interval of frequencies ω where the dispersion is of importance, i.e. the frequency range

$$v_F/L \ll \omega \ll \varepsilon_F/\hbar, \tag{1.10}$$

where v_F is the Fermi velocity, L is the length of the 1D conductor and ε_F is the Fermi energy. The problem we consider depends on three lengths, namely $a = 1/\kappa$ (the screening length), L and v_F/ω. We will consider in detail the cases with different relations between these lengths. All of the above-mentioned is related to a one-channel situation where the electrons of a single channel take part in the transport phenomena while the bottoms of other channels are above the Fermi level. In this paper we will also treat a more involved situation where several channels have bottoms below the Fermi level and contribute to the ac current. Their relative role in the transport will be discussed.

2. Single-channel conductance for a strong screening case

So far we have considered a dc response, keeping in mind the introduction of a concept of kinetic inductance and clarification of the corresponding physics. Obviously, the most important application of this concept is an ac conductance $G(\omega)$. In this section we will discuss a strong screening case $L \gg a$ where a part of the applied bias V sharply drops at the edges of the bridge, whereas the distribution function within the bridge is a superposition of the contributions emerging from both banks (cf. with the case treated in [6]):

$$f(\varepsilon) = \theta(v_x)f_0(\varepsilon - eV/2) + \theta(-v_x)f_0(\varepsilon + eV/2), \tag{2.1}$$

v_x being the velocity along the axis of the bridge. Equation (2.1) exploits the Liouville theorem, stating that the distribution function is constant along a trajectory. If the frequency ω of applied bias is so large that $\omega \gtrsim v_F/L$ the relation between J and $E = -\partial\varphi/\partial x$ is nonlocal. Namely, the electrons at a point x within the bridge have a memory of the bias at the moments $t - x/v_F$ and the charge distribution within the bridge is non-homogeneous. Under such circumstances the neutrality can also be violated.

To treat this situation and to visualize the physics we will begin with semiqualitative considerations. We will discuss a single-channel situation where the distribution is characterized by a single value of the Fermi velocity along the x direction. It is natural to expect that the applied voltage $V = V_0 \exp(-i\omega t)$ has three drops (see figure 1), i.e. $\tilde{V}/2$ corresponding to the left edge of the structure, $-EL$ (where $E = E_0 \exp(-i\omega t)$) corresponding to the channel where the electric field E_0 = const and the drop $\tilde{V}/2$ corresponding to the right edge. The neutrality condition implies that the drops at the edges are sharp—we will see below that their scale is determined by the screening length $a \equiv 1/\kappa$ where (ϵ being the dielectric constant)

$$\kappa^2 = \frac{8\pi e^2}{\mathcal{A}\pi\hbar\epsilon}\int_0^\infty dp_x\left(-\frac{\partial f_0}{\partial\varepsilon}\right) = \frac{8\pi e^2}{\mathcal{A}\pi\hbar\epsilon|v_F|} \tag{2.2}$$

[1] As indicated in [11] and [12] in a 1D sample of finite length a phase transition of a specific nature is possible. It takes place at the temperature $\Theta_c = 2\pi v_F^c/L$ determined by the strength of the e–e interaction. Here v_F^c is the renormalized value of the Fermi velocity with regard to this interaction. One can expect special behavior of $G(\omega)$ for the frequencies $\omega \sim \omega_c \equiv \Theta_c/\hbar$. However, for larger frequencies satisfying (1.9) one can disregard this phenomenon—see section 4 of paper [12]. The reasoning of this paper concerning the temperatures $T \gg \Theta_c$ is valid for the frequencies $\hbar\omega \gg \Theta_c = 2\pi v_F^c/L$ as well. It shows that the role of the e–e interaction for $\omega \gg 2\pi v_F^c/\hbar L$ is in replacement of $v_F \to v_F^c$.

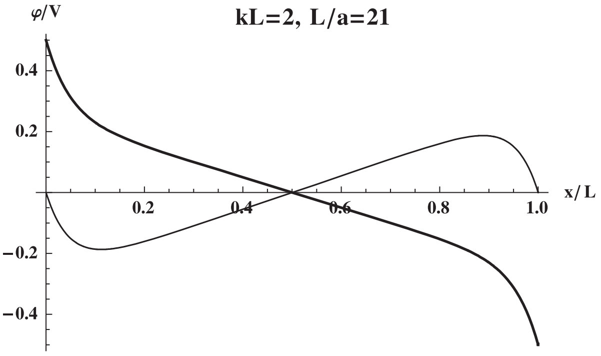

Figure 1. Potential distribution (the bold curve is the real part) along the bridge in a strong screening case. The potential has drops at the ends of the bridge.

while the electric field $E = E_0 \exp(-\mathrm{i}\omega t)$ is coordinate-independent within almost all the length L of the channel. Naturally, we have

$$V = \tilde{V} - EL. \tag{2.3}$$

The drops related to the edges of the channel (the edge perturbations) provide the contributions ξ_+ and ξ_- to the electron distribution function. In the linear approximation the electron trajectories are unperturbed by the electric field E within the structure and thus ξ_+ and ξ_- are given by standard solutions (cf [6])

$$\xi = \xi_+ + \xi_-,$$

$$\xi_+ = -\frac{e\tilde{V}}{2}\frac{\partial f_0}{\partial \varepsilon}\exp[-\mathrm{i}\omega(t - x/v_\mathrm{F})], \tag{2.4}$$

$$\xi_- = \frac{e\tilde{V}}{2}\frac{\partial f_0}{\partial \varepsilon}\exp[-\mathrm{i}\omega(t - (L-x)/v_\mathrm{F})].$$

In its turn, the homogeneous electric field E gives the following response of the electron distribution:

$$\eta_+ = \int_{t-x/v_\mathrm{F}}^{t} \mathrm{d}t' (\mathbf{v}e\mathbf{E})\frac{\partial f_0}{\partial \varepsilon}. \tag{2.5}$$

and

$$\eta_- = \int_{t-(L-x)/v_\mathrm{F}}^{t} \mathrm{d}t' (\mathbf{v}e\mathbf{E})\frac{\partial f_0}{\partial \varepsilon}. \tag{2.6}$$

Based on our assumptions we have

$$\eta_+ = \frac{(\mathbf{v}e\mathbf{E}_0)}{\mathrm{i}\omega}\frac{\partial f_0}{\partial \varepsilon}[\exp(-\mathrm{i}\omega(t - x/v_\mathrm{F})) - \exp(-\mathrm{i}\omega t)],$$

$$\eta_- = \frac{(\mathbf{v}e\mathbf{E}_0)}{\mathrm{i}\omega}\frac{\partial f_0}{\partial \varepsilon}[\exp(-\mathrm{i}\omega(t - (L-x)/v_\mathrm{F})) - \exp(-\mathrm{i}\omega t)]. \tag{2.7}$$

The product $(\mathbf{v}\mathbf{E}_0)$ for two different directions of $\mathbf{v}$ has different signs. For a homogeneous field the left-hand side of the Poisson equation vanishes. Thus with regard to equation (2.4) we have

$$\begin{aligned} e\frac{E_0 v_\mathrm{F}}{\mathrm{i}\omega}\frac{\partial f_0}{\partial \varepsilon}&[\exp(-\mathrm{i}\omega(t - x/v_\mathrm{F})) \\ &- \exp(-\mathrm{i}\omega(t-(L-x)/v_\mathrm{F}))] \\ = \frac{e\tilde{V}}{2}\frac{\partial f_0}{\partial \varepsilon}&[\exp(-\mathrm{i}\omega(t - x/v_\mathrm{F})) \\ &- \exp(-\mathrm{i}\omega(t-(L-x)/v_\mathrm{F}))] \end{aligned} \tag{2.8}$$

and

$$v_\mathrm{F} E_0 = \mathrm{i}\omega\frac{\tilde{V}}{2}, \qquad \text{or} \qquad E_0 = \mathrm{i}k\frac{\tilde{V}}{2} \quad \text{with } k = \frac{\omega}{v_\mathrm{F}}. \tag{2.9}$$

The total conduction current through the channel proportional to $v_\mathrm{F}(\xi_+ + \eta_+ - \xi_- - \eta_-)$ is

$$\begin{aligned} J = &\int_{p>0}\frac{\mathrm{d}p}{\pi\hbar}v_\mathrm{F}\frac{eE_0 v_\mathrm{F}}{\mathrm{i}\omega}\frac{\partial f_0}{\partial \varepsilon}\exp(-\mathrm{i}\omega t) \\ &\times[\exp(\mathrm{i}\omega x/v_\mathrm{F}) + \exp(\mathrm{i}\omega(L-x)/v_\mathrm{F}) - 2] \\ &-\int_{p>0}\frac{\mathrm{d}p}{\pi\hbar}\frac{e^2\tilde{V}}{2}v_\mathrm{F}\frac{\partial f_0}{\partial \varepsilon}\exp(-\mathrm{i}\omega t) \\ &\times[\exp(\mathrm{i}\omega x/v_\mathrm{F}) + \exp(\mathrm{i}\omega(L-x)/v_\mathrm{F})] \\ = &-\int_{p>0}\frac{\mathrm{d}p}{\pi\hbar}e^2 V v_\mathrm{F}\frac{\partial f_0}{\partial \varepsilon}\frac{1}{1-\mathrm{i}kL/2}\exp(-\mathrm{i}\omega t) \end{aligned} \tag{2.10}$$

where $k = \omega/v_\mathrm{F}$. We made use of the equation $V = \tilde{V}(1 - \mathrm{i}\omega L/v_\mathrm{F})$. The current is given by

$$J = G_0 V\frac{1}{1-\mathrm{i}kL/2}\exp(-\mathrm{i}\omega t). \tag{2.11}$$

For $kL \gg 1$ the impedance $Z(\omega)$ defined as

$$Z = \frac{V}{J} \tag{2.12}$$

is almost purely inductive.

3. Conductance for a weak screening case

In the opposite limit of weak screening $a \gg L$ the potential drops at the edges vanish and the electric field within a channel is equal to $E = V/L$. Only the terms with η contribute to the current J and the result is

$$\begin{aligned} J = &\int_{p>0}\frac{\mathrm{d}p}{\pi\hbar}v_\mathrm{F}\frac{v_\mathrm{F}e^2E}{\mathrm{i}\omega}\left(-\frac{\partial f_0}{\partial \varepsilon}\right)\exp(-\mathrm{i}\omega t) \\ &\times[\exp(\mathrm{i}\omega x/v_\mathrm{F}) + \exp(\mathrm{i}\omega(L-x)/v_\mathrm{F}) - 2]. \end{aligned} \tag{3.1}$$

In general the current depends on the coordinate x. This is feasible in the absence of screening. J is naturally the same at both contacts $x = 0$ and L. For the incoming and outgoing conduction current J_c we have

$$\frac{J_\mathrm{c}}{G_0 V} = \exp(\mathrm{i}kL/2)\frac{\sin kL/2}{kL/2}. \tag{3.2}$$

However, in this case (contrary to the previous one) the displacement current J_d related to the field discontinuity at the contacts is dominant. The displacement current is determined as

$$J_\mathrm{d} = -\mathrm{i}\frac{\omega\epsilon}{4\pi}\mathcal{A}E_\omega(0). \tag{3.3}$$

Therefore, for the total current $J = J_\mathrm{c} + J_\mathrm{d}$ we have

$$\frac{J}{G_0 V} = \left(-2i\frac{k}{\kappa^2 L} + \exp(\mathrm{i}kL/2)\frac{\sin kL/2}{kL/2}\right). \tag{3.4}$$

The impedance turns out to be capacitive. For $kL \gtrsim 1$ the first term in equation (3.4) is bigger than the second. However, the last term can be singled out as it oscillates as a function of external parameters.

This makes an essential difference with the case of complete neutrality where the impedance is always inductive, the current within the bridge is distributed homogeneously and this also holds for the electron kinetic energy density. The bridge indeed behaves as a single inductor.

4. Quantitative analysis of nonlocal response

The field can be written as (see appendix A)

$$E_\omega(x) = \frac{V}{L}\left(1 + \frac{\cosh\gamma(x-L/2) - (2/\gamma L)\sinh\gamma L/2}{\alpha - \cosh\gamma L/2 + (2/\gamma L)\sinh\gamma L/2}\right), \tag{4.1}$$

and the potential as

$$\varphi_\omega(x) = \frac{V}{2} \times \frac{(1-2x/L)[\alpha - \cosh\gamma L/2] + (2/\gamma L)\sinh\gamma(L/2-x)}{\alpha - \cosh\gamma L/2 + (2/\gamma L)\sinh\gamma L/2}. \tag{4.2}$$

The potential distribution along the bridge is given in figure 1. For the conduction current we get

$$\frac{J_\omega(x)}{G_0V} = \frac{\gamma^2}{\kappa^2} \times \frac{\tanh\gamma L/2 - \mathrm{i}(k/\gamma)(1-\cosh[\gamma(x-L/2)]/\cosh[\gamma L/2])}{(1-\mathrm{i}k\gamma^2L/2\kappa^2)\tanh\gamma L/2 - k^2L\gamma/2\kappa^2}. \tag{4.3}$$

For the total current using equation (3.3) we have

$$\frac{J_\omega}{G_0V} = (J_\omega(0) + J_\mathrm{d})/G_0V = \frac{\gamma^2}{\kappa^2}\frac{(1-k^2/\kappa^2)\tanh\gamma L/2 - \mathrm{i}k\gamma/\kappa^2}{(1-\mathrm{i}k\gamma^2L/2\kappa^2)\tanh\gamma L/2 - k^2L\gamma/2\kappa^2}. \tag{4.4}$$

In this case the (spatially dependent) conduction current is nearly uniform along the bridge. In the limit $a \ll L$, $ka \ll 1$ the current coincides with equation (2.11). The last inequality is equivalent to $k^2 \ll \kappa^2$ that can be rewritten as

$$\omega^2 \ll \omega_\mathrm{p}^2 \qquad \text{where } \omega_\mathrm{p}^2 = \frac{8e^2v_\mathrm{F}}{\mathcal{A}\hbar\epsilon}. \tag{4.5}$$

Now the 1D electron concentration n_1 is

$$n_1 = 2p_\mathrm{F}/\pi\hbar \qquad \text{where } p_\mathrm{F} = mv_\mathrm{F} \tag{4.6}$$

so that

$$\omega_\mathrm{p}^2 = \frac{4\pi n_1 e^2}{\mathcal{A}\epsilon m}, \qquad \text{or} \qquad \omega_\mathrm{p}^2 = \frac{4\pi n_3 e^2}{\epsilon m}. \tag{4.7}$$

ω_p can be interpreted as the plasma frequency. The phase of the response is determined from (we put $\tanh kL/2 = 1$)

$$\tan\phi = \frac{1-(ka)^2}{2/kL - (ka)\sqrt{1-(ka)^2}}. \tag{4.8}$$

The phase ϕ is $\pi/4$ for $kL = \omega L/v_\mathrm{F} \simeq 2$.

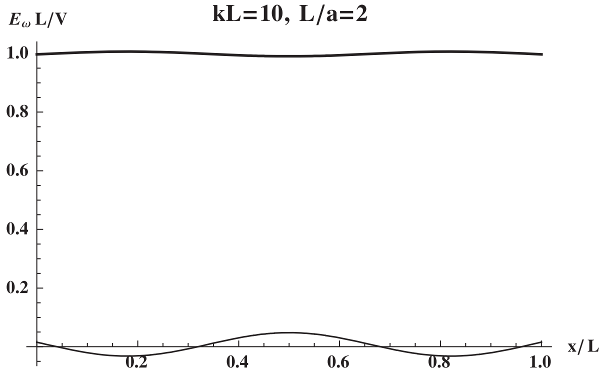

Figure 2. The real part of the field along the bridge in the case of weak screening is nearly uniform.

Let us turn to the opposite limiting case of weak screening. We assume $|v_\mathrm{F}|/\omega \leqslant L \leqslant a$. Now γ is given by

$$\gamma = \sqrt{k^2 - \kappa^2} \tag{4.9}$$

while the field is

$$E_\omega(x) = \frac{V}{L}\left(1 - \frac{\cos\gamma(x-L/2) - (2/\gamma L)\sin\gamma L/2}{\alpha + \cos\gamma L/2 - (2/\gamma L)\sin\gamma L/2}\right). \tag{4.10}$$

In this case, the dominant real part of the field is weakly dependent on the coordinate (see figure 2). The potential is given by

$$\varphi_\omega(x) = \frac{V}{2} \times \frac{(1-2x/L)[\alpha + \cos\gamma L/2] + (2/\gamma L)\sin\gamma(L/2-x)}{\alpha + \cos\gamma L/2 - (2/\gamma L)\sin\gamma L/2}. \tag{4.11}$$

We get for the conduction current

$$\frac{J_\omega(x)}{G_0V} = \frac{\gamma^2}{\kappa^2} \times \frac{\tan\gamma L/2 + \mathrm{i}(k/\gamma)(1-\cos[\gamma(x-L/2)]/\cos[\gamma L/2])}{-(1+\mathrm{i}k\gamma^2L/2\kappa^2)\tan\gamma L/2 + k^2L\gamma/2\kappa^2}. \tag{4.12}$$

As is seen in figure 3 the conduction current in this case oscillates along the bridge. To determine the total current let us consider the conduction current at the ends of the bridge:

$$\frac{J_\omega(0)}{G_0V} = \frac{\sin\gamma L/2}{\gamma L/2}\{(1+\kappa^2/\gamma^2)\cos\gamma L/2 - (\mathrm{i}\sqrt{1+\kappa^2/\gamma^2} + 2\kappa^2/\gamma^3L)\sin\gamma L/2\}^{-1}. \tag{4.13}$$

The total current is given by

$$\frac{J_\omega}{G_0V} = \frac{\gamma^2}{\kappa^2}\frac{(1-k^2/\kappa^2)\tan\gamma L/2 - \mathrm{i}k\gamma/\kappa^2}{k^2L\gamma/2\kappa^2 - (1+\mathrm{i}k\gamma^2L/2\kappa^2)\tan\gamma L/2}. \tag{4.14}$$

The real part of the conductance vanishes provided $\sin\gamma L/2 = 0$, i.e. for frequencies

$$\omega = \omega_\mathrm{p}\sqrt{1+(2\pi na/L)^2},$$

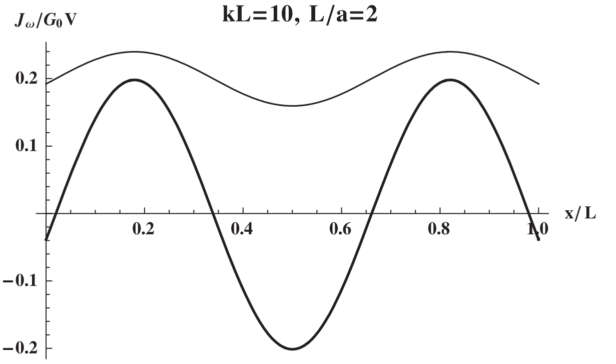

Figure 3. Conduction current distribution along the ballistic bridge.

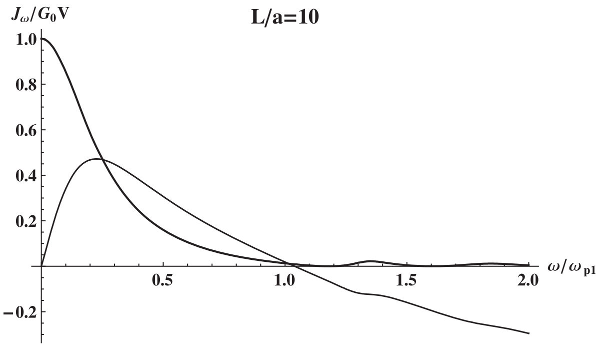

Figure 4. Total current as a function of frequency.

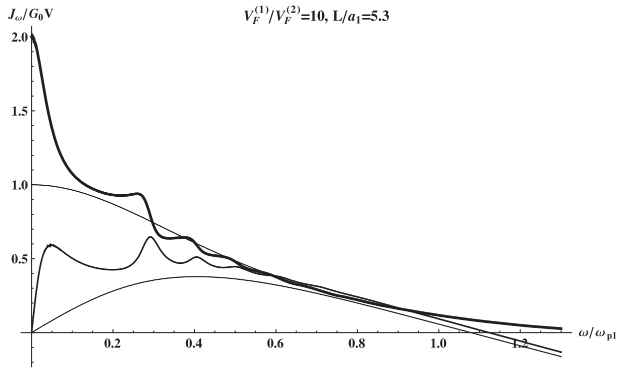

Figure 5. Total current for two channels.

the response is of a purely reactive nature. For $\kappa^2 \ll k^2$ the conduction current and the total current are given by equation (3.2) and equation (3.4), respectively. The last equation can be presented in the form

$$J = J_c + J_d = \frac{G_0 V}{kL/2}\left[-\mathrm{i}\left(\frac{k}{\kappa}\right)^2 + \mathrm{e}^{\mathrm{i}kL/2}\sin\frac{kL}{2}\right]. \quad (4.15)$$

The second term is much smaller than the first one. We believe, however, that the oscillating term can be observed provided a first or second derivative of the current over ω or v_F is measured. The current dependence on the frequency $\omega/\omega_p \equiv k/\kappa$ is given in figure 4. One can see that there is an interval of frequencies where the inductive contribution to the current dominates.

5. Multi-channel conductance for strong screening

So far we have considered a single-channel situation with a given value of the velocity v_F. One can treat a multi-channel case in the same manner, still assuming the overall neutrality. Our reasoning will be based on the assumption that the electrons in both reservoirs are in equilibrium at the same temperature, the difference between their chemical potentials being equal to eV. In this situation one has different Fermi velocities $v_F^{(n)}$ for different channels. The oscillating functions ξ and η are also different for different channels and the only way to support the neutrality is to have neutrality conditions for each channel separately. The potential drops at the edges are still abrupt and determined by a. However, the relations between the potential drops $\tilde{V}$ and field E_0 are different for different channels. While all these drops are related to electrochemical potentials, it means that the channels are characterized by some *partial* chemical potentials different for different channels. It is not an unusual case for a non-equilibrium situation. In this way the overall charge neutrality can be maintained. Having these considerations in mind, we will have for a multi-channel case

$$\frac{G(\omega)}{NG_0} = \frac{1}{\mathcal{N}}\sum_{n=1}^{n=\mathcal{N}} \frac{1}{1 - \mathrm{i}k_n L/2}. \quad (5.1)$$

Correspondingly, for $kL \gg 1$ we have a purely inductive response, the total inductance being a sum over the channels.

This contribution should be compared with the geometric inductance $\mathcal{L}_g$. We have

$$\mathcal{L}_g = 2L \ln \frac{L}{\sqrt{\mathcal{A}}}. \quad (5.2)$$

Omitting the logarithmic factor one can write the following inequality for the preponderance of reactive resistance as compared to the geometric one:

$$c^2/Gv_F \gg 1. \quad (5.3)$$

It can be easily satisfied.

Now let us concentrate on a two-channel case. First we consider the case where the number of electrons within the second channel is relatively small, so that the Fermi velocity of this channel $v_F^{(2)}$ is much less than the velocity of the first channel $v_F^{(1)}$. We have $k_1/k_2 = v_F^{(2)}/v_F^{(1)} \ll 1$. The total current is presented in figure 5. We are interested in the region of relatively small frequencies $\omega \lesssim \kappa v_F^{(2)} \ll \kappa v_F^{(1)}$ since for bigger frequencies the response is always capacitive and coincides with the result for a one-channel case (this is illustrated by the thin curves in the figure). In this case the imaginary contribution to the current is always smaller than the real one.

If the Fermi velocities of the two channels get closer the imaginary inductive contribution to the current can be larger than the real one. Then both channels are important and the situation is illustrated in figure 6. The quality factor can be larger for two channels than for a single channel, as is seen in figure 7.

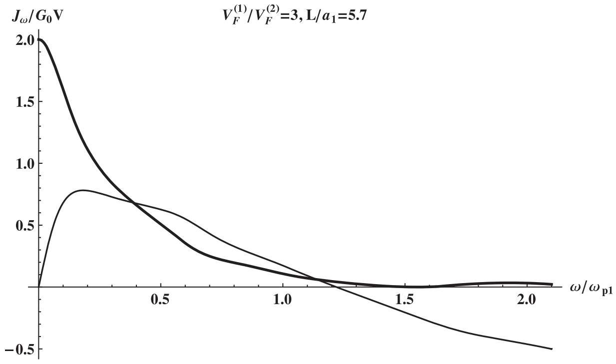

Figure 6. Total current for the case where both channels are important.

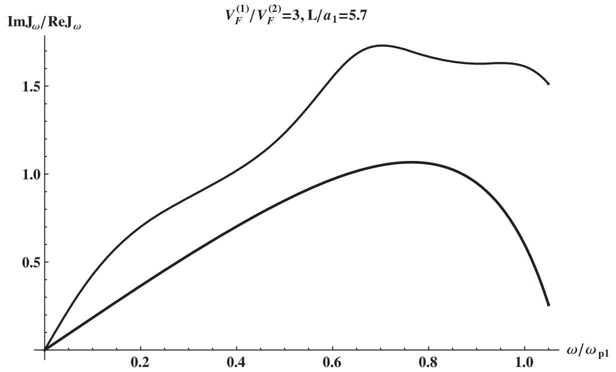

Figure 7. Quality factors for single channel and two channels.

6. Conclusion

For the treatment of our problem we used the Boltzmann equation for a one-particle electron distribution function. As is well known, a single-channel state is unstable and, as a result, one gets the so-called Tomonaga–Luttinger (T–L) liquid [9, 10]. In [7] $G(\omega)$ was calculated within T–L liquid theory. For our purpose it is important that, for a weak electron–electron interaction, in particular, for a small gas parameter

$$g \equiv \frac{e^2}{\pi \epsilon \hbar v_{\rm F}}, \tag{6.1}$$

ϵ being the dielectric susceptibility, the results turned out the same as within the Fermi gas approach. For $g \gtrsim 1$ the only difference appears to be a replacement of $v_{\rm F}$ by its renormalized value $v_{\rm F}^c$. Having this in mind, we contented ourselves with application of the Boltzmann equation for the electron Fermi gas to treat our problem. The most essential difference between [7] and the results of this paper is the following. For $g \ll 1$ and $k/\kappa \ll 1$ we get equation (2.11) whereas no such result is present in [7] under these conditions because of a different electrodynamics. The kinetic inductance (in general, kinetic complex resistance) can be registered in a standard way by any phase measurement, in particular by a measurement of the impedance of a contour including the bridge and capacitor in parallel. According to our estimates, the inductance can be readily made larger than the geometric inductance and thus the properties of the circuit are insensitive to the geometry. Then, $\mathcal{L}_{\rm k}$ can be manipulated by variation of the bridge parameters with the help of a split gate. The kinetic inductance discussed above allows us to introduce relatively easily inductive elements to nanoelectronic devices that can, in principle, allow us to realize phase sweeping blocks. These structures in the ballistic electron transport regime may find applications in phase sweeping electronic devices, working in the high frequency range. For instance, a feedback in active or passive electronic devices, when the phase relationships can be as important as the amplitude ones, can be provided by a nanobridge.

Acknowledgment

The authors are grateful to V V Afonin for a valuable discussion.

Appendix A. Treatment of a single-channel case

We will give a detailed calculation of the fields and currents. We begin with the Boltzmann equation for a single-channel situation:

$$\frac{\mathrm{d}}{\mathrm{d}t} f = -\frac{f - f_0}{\tau}, \tag{A.1}$$

where

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + eE\frac{\partial}{\partial p_x} \tag{A.2}$$

and τ is the time of relaxation (which would not enter the final formulae). The solution of this equation by the method of characteristics can be written in the form

$$f(x, p, t) = \int_{-\infty}^{t} \frac{\mathrm{d}t'}{\tau} e^{(t'-t)/\tau} f_0(\varepsilon[p_x(t')]), \tag{A.3}$$

where the integration goes over the trajectory determined from the equations of motion:

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \mathbf{v}, \qquad \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = e\mathbf{E}(\mathbf{r}, t) \tag{A.4}$$

so that $p_x(t')|_{t'=t} = p_x$, $x(t')|_{t'=t} = x$. The trajectory can be written explicitly as

$$p_x(t') = p_x + \int_t^{t'} \mathrm{d}t_1\, eE(x(t_1), t_1), \tag{A.5}$$

$$x(t') = x + \frac{p_x}{m}(t'-t) + \int_t^{t'} \mathrm{d}t_1 \int_t^{t_1} \mathrm{d}t_2\, eE(x(t_2), t_2). \tag{A.6}$$

Therefore equation (A.3) can be written as

$$f(x, p, t) = \int_{-\infty}^{t} \frac{\mathrm{d}t'}{\tau} e^{(t'-t)/\tau} f_0\Bigg(\varepsilon + \frac{p_x}{m}\int_t^{t'} eE(x(t_1), t_1)\,\mathrm{d}t_1 + \frac{1}{2m}\left(\int_t^{t'} eE(x(t_1), t_1)\mathrm{d}t_1\right)^2\Bigg), \tag{A.7}$$

or in the linear approximation with respect to the field

$$f(x, p, t) = \int_{-\infty}^{t} \frac{\mathrm{d}t'}{\tau} e^{(t'-t)/\tau} \times \left(f_0(\varepsilon) + \frac{\partial f_0}{\partial \varepsilon} v_x \int_t^{t'} eE(x(t_1), t_1)\,\mathrm{d}t_1\right). \tag{A.8}$$

Now in the last equation the free motion trajectory $x(t_1) = x+(p_x/m)(t_1-t)$ can be used. Shifting the integration variable $t' = t+\xi$ and integrating by parts we get for $\Delta f = f - f_0$

$$\Delta f = -ev_x \frac{\partial f_0}{\partial \varepsilon}\int_{-\infty}^{0} \mathrm{d}\xi\, e^{\xi/\tau} E(x+v_x\xi, t+\xi). \tag{A.9}$$

Now we can put $\tau \to \infty$, considering the ballistic case. Then we get

$$\Delta f = -ev_x \frac{\partial f_0}{\partial \varepsilon}\int_{-\infty}^{0} \mathrm{d}\xi\, E(x+v_x\xi, t+\xi). \tag{A.10}$$

We assume as the boundary conditions that the field vanishes for $E < 0$ and $E > L$. This means for the trajectory coming from the left bank $v_x > 0$ that the integration is from $-x/v_x$ to 0:

$$\theta(v_x)\int_{-x/v_x}^{0} \mathrm{d}\xi\, E(x+v_x\xi, t+\xi), \tag{A.11}$$

and for the trajectory coming from the right bank with velocity $v_x < 0$ the integration is from $(L-x)/v_x$ to 0:

$$\theta(-v_x)\int_{(L-x)/v_x}^{0} \mathrm{d}\xi\, E(x+v_x\xi, t+\xi). \tag{A.12}$$

The result is

$$\Delta f = -\theta(v_x)\frac{\partial f_0}{\partial \varepsilon}\int_0^x \mathrm{d}z\, eE(z, t+(z-x)/v_x) - \theta(-v_x)\frac{\partial f_0}{\partial \varepsilon}\int_L^x \mathrm{d}z\, eE(z, t+(x-z)/|v_x|). \tag{A.13}$$

We insert this solution into the Poisson equation and get

$$\frac{2}{\kappa^2}\frac{\mathrm{d}E}{\mathrm{d}x} = \int_0^x \mathrm{d}z\, E(z, t+(z-x)/|v_\mathrm{F}|) - \int_x^L \mathrm{d}z\, E(z, t+(x-z)/|v_\mathrm{F}|), \tag{A.14}$$

Introducing the notation $k = \omega/|v_\mathrm{F}|$ we get the equation for the field

$$\frac{2}{\kappa^2}\frac{\mathrm{d}E_\omega}{\mathrm{d}x} = -\mathrm{e}^{-\mathrm{i}kx}\int_0^L \mathrm{d}z\, E_\omega(z)\mathrm{e}^{\mathrm{i}kz} + 2\int_0^x \mathrm{d}z\, E_\omega(z)\cos k(x-z). \tag{A.15}$$

The conduction current is determined from

$$J_\omega(x)/G_0 = \mathrm{e}^{-\mathrm{i}kx}\int_0^L \mathrm{d}z\, E_\omega(z)\mathrm{e}^{\mathrm{i}kz} + 2\mathrm{i}\int_0^x \mathrm{d}z\, E_\omega(z)\sin k(x-z), \tag{A.16}$$

where $G_0 = e^2/\pi\hbar$. First we consider a strong screening case. Therefore we assume $a \leqslant |v_\mathrm{F}|/\omega \leqslant L$. The Laplace transform of equation (A.15) is given by

$$\frac{2}{\kappa^2}(pE_\mathrm{p} - E_\omega(0)) = -\frac{A}{p+\mathrm{i}k} + 2\frac{pE_\mathrm{p}}{p^2+k^2}, \tag{A.17}$$

where

$$A = \int_0^L \mathrm{d}z\, E_\omega(z)\mathrm{e}^{\mathrm{i}kz}. \tag{A.18}$$

The original is

$$E_\omega(x) = E_\omega(0)\left(1 - \frac{\kappa^2}{\gamma^2}(1-\cosh\gamma x)\right) - \frac{A\kappa^2}{2}\left(\frac{\sinh\gamma x}{\gamma} + \frac{\mathrm{i}k}{\gamma^2}(1-\cosh\gamma x)\right), \tag{A.19}$$

where

$$\gamma^2 = \kappa^2 - k^2. \tag{A.20}$$

Using this relation in equation (A.18) we determine A:

$$A = 2E_\omega(0)\frac{\sinh\gamma L/2}{\gamma\cosh\gamma L/2 - \mathrm{i}k\sinh\gamma L/2}. \tag{A.21}$$

Then for the field we get

$$E_\omega(x) = E_\omega(0)\left(1 + \frac{\cosh\gamma(x-L/2) - \cosh\gamma L/2}{\alpha}\right), \tag{A.22}$$

where

$$\alpha = \frac{\gamma^2}{\kappa^2}(\cosh\gamma L/2 - (\mathrm{i}k/\gamma)\sinh\gamma L/2). \tag{A.23}$$

For the potential we have

$$\varphi_\omega(x) = -E_\omega(0)\left(\left(1 - \frac{\cosh\gamma L/2}{\alpha}\right)(x-L/2) + \frac{\sinh\gamma(x-L/2)}{\alpha\gamma}\right). \tag{A.24}$$

The boundary conditions imposed on the potential $\varphi_\omega(0) = V/2$ and $\varphi_\omega(L) = -V/2$ yield

$$E_\omega(0) = \frac{V}{L}\frac{\alpha}{\alpha - \cosh\gamma L/2 + 2(\sinh\gamma L/2)/\gamma L}. \tag{A.25}$$

Finally, the field can be written as

$$E_\omega(x) = \frac{V}{L}\left(1 + \frac{\cosh\gamma(x-L/2) - (2/\gamma L)\sinh\gamma L/2}{\alpha - \cosh\gamma L/2 + (2/\gamma L)\sinh\gamma L/2}\right), \tag{A.26}$$

and the potential as

$$\varphi_\omega(x) = \frac{V}{2} \times \frac{(1-2x/L)[\alpha - \cosh\gamma L/2] + (2/\gamma L)\sinh\gamma(L/2-x)}{\alpha - \cosh\gamma L/2 + (2/\gamma L)\sinh\gamma L/2}. \tag{A.27}$$

Appendix B. Screening in 1D nanostructures

We consider a 1D semiconducting structure surrounded by a good conductor. Let us assume that there is an external perturbation of the charge density:

$$\rho^{(e)} \propto \mathrm{e}^{\mathrm{i}kx} \tag{B.1}$$

that brings about an electrostatic potential:

$$\varphi \propto \frac{1}{k^2}. \tag{B.2}$$

This is a would-be potential in dielectrics. The total potential in semiconductors includes a part due to spatial redistribution of conduction electrons, i.e. the effect of screening. It can be found from

$$\epsilon\nabla^2\varphi = -4\pi\rho, \tag{B.3}$$

where ϵ is the dielectric susceptibility while ρ is the total electron density $\rho = \rho^{(e)} + \rho^{(i)}$, including the part

$$\rho^{(i)} \equiv \frac{\partial n_3}{\partial\mu}e\varphi$$

due to conduction electrons' redistribution in the field φ (it is linear in φ provided this quantity can be considered as small). We are going to investigate this effect of screening in more detail. We have

$$\nabla^2\varphi \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) + \frac{\partial^2\varphi}{\partial x^2} \equiv \frac{\mathrm{d}^2\varphi}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\varphi}{\mathrm{d}r} - k^2\varphi \tag{B.4}$$

$$\varphi|_{r=b-0} = \varphi|_{r=b+0}, \qquad \left.\frac{\partial\varphi}{\partial r}\right|_{r=b-0} = \left.\frac{\partial\varphi}{\partial r}\right|_{r=b+0}, \tag{B.5}$$

where b is the radius of a 1D conductor circular cross section. Now we assume that $\rho^{(e)}$ has the form

$$\rho^{(e)} = \mathrm{e}^{\mathrm{i}kx} f(r)\rho^{(0)}, \tag{B.6}$$

where $\rho^{(0)}$ determines the scale of the charge density variation due to external sources. The actual form of $f(r)$ is immaterial for analysis of a 1D screening provided $f(r)$ vanishes for $r \geqslant b$ and is a smooth function for $r \leqslant b$. Now we have

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\varphi}{\mathrm{d}r} - \lambda^2\varphi = -\frac{4\pi}{\epsilon}\rho^{(0)} f(r) \qquad \text{for } r \leqslant b,$$
$$\lambda^2 = k^2 + \kappa^2, \quad \kappa^2 = \frac{4\pi e^2}{\epsilon}\frac{\partial n_3}{\partial\mu}, \tag{B.7}$$

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\varphi}{\mathrm{d}r} - k^2\varphi = 0 \qquad \text{for } r \geqslant b. \tag{B.8}$$

In what follows it is convenient to take $f(r)$ in the form

$$f(r) = \frac{\epsilon}{4\pi\rho^{(0)}}AI_0(\beta r) \qquad \text{for } r \leqslant b \quad \text{where } \beta \ll 1/b,$$
$$f(r) = 0 \qquad \text{for } r \geqslant b.$$

Here A is a constant and $I_0(z)$ is the modified Bessel function. Equation (B.7) takes the form

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\varphi}{\mathrm{d}r} - \lambda^2\varphi = -AI_0(\beta r). \tag{B.9}$$

A finite general solution for $r \leqslant b$ can be presented as

$$\varphi = C_1 I_0(\lambda r) + \frac{1}{\lambda^2 - \beta^2}AI_0(\beta r) \tag{B.10}$$

while for $r \geqslant b$ it is

$$\varphi = C_2 K_0(kr) \tag{B.11}$$

where $C_{1,2}$ are constants and $K_0(z)$ is the McDonald function. As $\beta b \ll 1$ we can expand $I_0(\beta r)$ (see below). Making use of boundary conditions (B.5) one gets

$$C_1 I_0(\lambda b) + \frac{A}{\lambda^2 - \beta^2} = C_2 K_0(kb),$$
$$C_1\lambda I_0'(\lambda b) + \frac{A/2}{\lambda^2 - \beta^2}\beta^2 b = C_2 k K_0'(kb). \tag{B.12}$$

We have for $y \ll 1$

$$I_0(y) = 1 + y^2/4,\ K_0(y) = -\ln(y/2)I_0(y) + \psi(1), \tag{B.13}$$

where ψ is the digamma function whereas for $y \gg 1$

$$I_0(y) = \frac{1}{\sqrt{2\pi y}}\mathrm{e}^y. \tag{B.14}$$

Now, $kb \ll 1$ and $\beta^2 \ll \lambda^2$. System (B.12) can be simplified as

$$C_1 I_0(\lambda b) + \frac{A}{\lambda^2} = C_2 \ln\frac{1}{kb},$$
$$C_1\lambda b I_0'(\lambda b) + \frac{A}{2\lambda^2}(\beta b)^2 = -C_2, \tag{B.15}$$

so that

$$C_1 = -\frac{1}{\left[I_0(\lambda b) + (\lambda b)I_0'(\lambda b)\ln(1/kb)\right]}\frac{A}{\lambda^2},$$
$$C_2 = \frac{(\lambda b)I_0'(\lambda b)}{\left[I_0(\lambda b) + (\lambda b)I_0'(\lambda b)\ln(1/kb)\right]}\frac{A}{\lambda^2}. \tag{B.16}$$

$\varphi(r)$ for $r \leqslant b$ can be presented in the following way:

$$\varphi(r) = \left[1 - \frac{I_0(\lambda r)}{I_0(\lambda b) + (\lambda b)I_0'(\lambda b)\ln(1/kb)}\right]\frac{A}{\lambda^2}. \tag{B.17}$$

Now we will treat two limiting cases. For

$$\lambda b \ll 1 \tag{B.18}$$

we get

$$\varphi(r) = \frac{2b^2\ln(1/kb) - r^2}{4}A. \tag{B.19}$$

(Here we have considered the quantity $\ln(1/kb)$ as big.) This means that the screening is determined essentially by the radius b, i.e. by the lateral dimensions of the conductor. In the opposite case

$$\lambda b \gg 1 \tag{B.20}$$

we have

$$\varphi(r) = \frac{A}{\kappa^2 + k^2} \tag{B.21}$$

instead of $\varphi = A/k^2$ in an isolator. The screening is determined by $a \equiv 1/\kappa$.

References

[1] Sharvin Yu V 1965 *Zh. Eksp. Teor. Fiz.* **48** 984
Sharvin Yu V 1965 *JETP* **21** 655 (Engl. Transl.)

[2] Yanson I K 1983 *Fiz. Nizk. Temp.* **9** 676
Yanson I K 1983 *Sov. J. Low Temp. Phys.* **9** 343 (Engl. Transl.)

[3] Landauer R 1970 *Phil. Mag.* **21** 863
Büttiker M, Imry Y, Landauer R and Pinhas S 1985 *Phys. Rev.* B **31** 6207
[4] Imry Y 1986 *Directions in Condensed Matter Physics* ed G Grinstein and E Mazenko (Singapore: World Scientific)
[5] Beenakker C W J and van Houten H 1990 *Electronic Properties of Multilayers and Low dimensional Semiconductor Structures* (New York: Plenum)
[6] Kulik I O, Omel'yanchuk A N and Tuluzov I G 1982 *Sov. J. Low Temp. Phys.* **8** 386
[7] Sablikov V A and Shchamkhalova B S 1998 *Pis. Zh. Eksp. Teor. Fiz.* **67** 184
Sablikov V A and Shchamkhalova B S 1998 *JETP Lett.* **67** 196
Sablikov V A and Shchamkhalova B S 1998 *Phys. Rev.* B **58** 13847
[8] Ponomarenko V V 1996 *Phys. Rev.* B **54** 10328
[9] Tomonaga S 1950 *Prog. Theor. Phys.* **5** 544
[10] Luttinger J M 1963 *J. Math. Phys.* **4** 1154
[11] Afonin V V and Petrov V Yu 2008 *JETP* **11** 542
[12] Afonin V V, Gurevich V L and Petrov V Yu 2009 *JETP* **108** 845